Synopsis for Ph.D Programme

Title of Research

A study on multi-number system and its use to develop metric and norm structures in multiset context.

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1. Introduction:

Mutisets (Bag) are of interest in a certain area of mathematics, computer science, physics and philosophy. A multiset is a collection of objects (called elements) in which elements may occur more than once. The number of times an element occurs in a multiset is called its multiplicity. The cardinality of a multiset is the sum of the multiplicities of its elements. Whether in classical set theory, a set is a well-defined collection of distinct objects. Therefore, a set is a multiset in which distinct elements occur only once. In various counting argument, it is convenient to distinguish between a set like {a,b,c} and a collection like {a,a,a,b,c,c}. The latter, if viewed as a set, will be identical to the former. However, it has some of its elements purposely listed several times. Therefore, cardinality of a multiset assumes 'plurality without diversity' whereas classical mathematics accepts the principal of plurality without diversity. In algebra, for example, repeated roots of polynomials, although identical in all respect, are treated as multiplicity. Also, repeated observations in a statistical sample are counted as a multiplicity. We formalize it by defining a multiset as a collection of elements, each considered with certain multiplicity. For the sake of convenience, a multiset is written as $\{k_1/x_1, k_2/x_2,\ldots, k_n/x_n\}$ in which the element x_i occurs $k_i$times. We observe that each multiplicity k_i is a positive integer. Classical set theory states that a given element can appear only once in a set, it assumes that all mathematical objects occur without repetition. Thus, there is only one number four, one field of complex numbers, etc. So, the only possible relation between two mathematical objects is either they are equal or they are different. However, in the physical world there is enormous repetition. For instance, there are many hydrogen atoms, many water molecules, and many strands of DNA etc. The concept of distance is very basic in mathematics and it has wide applications in different field.

2. Brief History of previous work done:

The word 'multiset' (mset, for short) which abbreviates the term 'multiple-membership set', is now the commonly accepted name for this concept, replacing 'bag', 'bunch', 'weighted set', 'occurrence set', 'heap', 'sample', and 'fireset' – finitely repeated element set. D. Blizard first studied multiset in 1989; he made a thorough study of multiset theory, real valued multisets and negative membership of the

elements of multisets ([1], [2], [3], [4]). K.P. Girish and S. J. John introduced and studied the concept of multiset topologies, multiset relations, multiset functions, chains and antichains of partially ordered multisets ([9], [10], [11], [12], [13]). D. Tokat studied the concept of soft multi continuous function ([20]). Concept of multi groups and soft multi groups are found in the studies of Sk. Nazmul and S. K. Samanta ([17], [18]). Many other authors like Chakraborty et al. ([5], [6], [7], [8]), S.P. Jena et al. ([14]), J.L. Peterson ([19]) also studied various properties and applications of multisets.

In 2012, Girish and John ([11]) introduced multiset topologies induced by multiset relations. The same author further studied the notions of open sets, closed sets, basis, sub basis, closure and interior, continuity and related properties in M-topological spaces in [12]. The concept of semi open, semi closed multisets was introduced by J. Mahanta and D. Das in 2014 in [15], which were then used to study semi compactness in multiset topology. Further the concept of exterior and boundary in multiset topological space were given by the same authors in the year 2015 in [16].

3. Proposed plan of work to be done:

From the research articles surveyed above, it appears that there is a lot of scope of future research works in the field of multi set. Especially the further study can be carried out in the following directions:

I would like to concentrate my Ph.D. research work

- To develop a 'Multi Natural Number System' and to study on the algebraic operations and order relations defined on it.
- To study the possible extension process of Multi Natural Number System towards Multi Integer System, Multi Rational Number System, Multi Real Number System etc. Also, to study thoroughly the algebraic operations and order relations defined on them.
- To study algebraic and topological structures of Multi Number System.
- To study metric structure, norm structure in multi setting using multi number system.

4. Work already done:

Developments of Multi Number System have already been started. Some work has been done in defining Multi Natural Number and some properties of these have been studied. A research paper will be communicated to a journal shortly for possible publication.

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